Econ 413R: Computational Economics

Spring Term 2013

Supplemental Handout: OLG Models in a DSGE context

1 Overlapping Generations Models

Our dyanmic programming approach to the household's problem cannot be applied to an overlapping generations (OLG) model. This is because the value-function for an n-year-old agent is different from that of an $n + 1$-year-old agent. However, in our infintely-lived agent models from sections 2 through 4, the problem is the same since the agent still has an infintely number of periods to live every period.

Despite this drawback, we can still express an OLG model in the same notation.

Consider, for example, the Euler equations, for an OLG model with N-

period-lived agents.

$$\begin{aligned}
u_c(c_{1t}) &= \beta E\{u_c(c_{2,t+1})(1 + r_{t+1} - \delta)\} \\
u_c(c_{2t}) &= \beta E\{u_c(c_{3,t+1})(1 + r_{t+1} - \delta)\} \\
&\vdots \\
u_c(c_{N-1,t}) &= \beta E\{u_c(c_{N,t+1})(1 + r_{t+1} - \delta)\}
\end{aligned} \tag{1.1}$$

There are also a set of budget contraints, one for each agent, that define the consumptions, $\{c_{nt}\}_{n=1}^{N}$.

$$\begin{aligned}
c_{1t} &= w_t \ell_{1t} - k_{2,t+1} \\
c_{2t} &= w_t \ell_{2t} + (1 + r_t - \delta)k_{2t} - k_{3,t+1} \\
&\vdots \\
c_{N-1,t} &= w_t \ell_{N-1,t} + (1 + r_t - \delta)k_{N-1,t} - k_{N,t+1} \\
c_{N,t} &= w_t \ell_{N,t} + (1 + r_t - \delta)k_{N,t}
\end{aligned} \tag{1.2}$$

Aggregate capital (K) and labor (L) will be the sums of the ks and ℓs over all cohorts.

$$\begin{aligned}
K_t &= \sum_{n=2}^{N} k_{nt} \\
L_t &= \sum_{n=1}^{N} \ell_{nt}
\end{aligned} \tag{1.3}$$

Wages and interest rates are defined from first-order conditions for firms and will be conditions similar to (3.9) and (3.10).

$$\begin{aligned} r_t &= f_K(K_t, L_t, z_t) \\ w_t &= f_L(K_t, L_t, z_t) \end{aligned} \tag{1.4}$$

Finally, we have a law of motion for the exogenous productivity shock.

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{1.5}$$

Equations (1.1) - (1.5) are a dynamic system of $2N + 4$ equations and variables. We can categorize our variables as before.

$$\begin{aligned} X_t &= (\{k_{n,t+1}\}_{n=2}^{N}) \\ Y_t &= (\{c_{n,t}\}_{n=1}^{N}, K_t, L_t, r_t, w_t) \\ Z_t &= z_t \end{aligned} \tag{1.6}$$

We can solve and simulate this system in exactly the same way we do those from sections 2 through 4 as explained in the next chapter.