Exercises

Homework 1

Using the methods discussed in section 3.1.1, write a recursive form of the infinite sum, S_t.

Derive the Euler equation.

Homework 2

Using the methods discussed in section 3.1.2, write a recursive form of the infinite sum, D_t.

Homework 3

Using the same setup as Problem 14 from the Week 3 DSGE homework, let's tweak the utility function a little bit by including a preference shock to consumption Lester, Pries, and Sims (2013).

$$u(c_t, \ell_t) = \nu_t \frac{c_t^{1-\gamma}}{1-\gamma} + a\frac{(1-\ell_t)^{1-\gamma}}{1-\gamma}$$

where $\nu_t = (1-\rho_\nu) + \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$; with $\varepsilon_t^\nu \sim \text{i.i.d.}(0, \sigma_\nu^2)$ and $\sigma_\nu = .02$.

nu_t is a multiplicative shock to utility from consumption and will therefore shock marginal utility of consumption as well. It is like a demand shock. Note that the coefficient of relative risk aversion and the elasticity of substitution are independent of this shock.

Also assume $\varepsilon_t^z \sim \text{i.i.d.}(0, \sigma_z^2)$ and $\sigma_z = .01$.

Rewrite the two characterizing equations as they should now appear with this preference shock to consumption.

Write the Dynare code to perform a stochastic simulation of this model for 2100 periods, use the second-order Taylor-series approximation, and generate IRFs of all the variables to each shock for 100 periods. Provide the output, graphs and code as part of your homework submission. Comment on the IRF graphs; intuitively describe the responses of output and consumption each of the shocks.

Homework 4

Modify the Dynare code used above, eliminating the stochastic simulation, and gearing it toward a Bayesian estimation of the following parameters: $\gamma, a, \beta, \rho_z, \rho_\nu, \sigma^z$ and σ_ν. You will need to include new variables in your model that are more suited toward Bayesian estimation:

$$dc = c_t - c_{t-1}$$

$$dy = y_t - y_{t-1}$$

These will be the variables you will be observing in the code. I will provide the data file for you. Start your estimation according to the table below.[1]

Do this estimation for 20000 replications, 3 Metropolis Hastings blocks, drop the first 15% of the replications (burn in of 15%), start `mh_jscale` at 0.5 and ADJUST the `mh_jscale` as you monitor the acceptance rate so that you get an acceptance rate as close to 25% as possible once the replications within

[1] Prior distributions inspired by the in class example and Del Negro, Schorfheide, Smets and Wouters, 2005

Table 1: Bayesian Estimation Setup

Parameter	**Prior Distribution**	**Prior Mean**	**Prior St. Dev.**
γ	Gamma	2.5	0.05
a	Beta	0.5	0.01
β	Beta	0.98	0.002
ρ_z	Beta	0.90	0.05
ρ_ν	Beta	0.95	0.05
σ_z	Inverse Gamma	0.01	∞
σ_ν	Inverse Gamma	0.02	∞

the block seem to stabilize around a particular rate. Report the posterior mode for each of the parameters. This is the most important information. Also report the posterior mean and confidence intervals for each parameter from the provided output. Provide the graphs from the output.

If you were to redo the Bayesian estimation, for which parameters would you re-specify the prior distribution or how would you modify the replication procedure? Use the graphs as support for your answer. If you feel the priors were reasonably specified, provide support (again from the graphs) toward the legitimacy of the priors specified.

References

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Figure 1: Sample Dynare Model File