# Econ 413R: Computational Economics

Spring Term 2013

# Dynamic Stochastic General Equilibrium Modeling Homework Set

Week 2

## Homework 1

For the Brock and Mirman model, find the value of $A$ in the policy function. Show that your value is correct.

For this case find an algebraic solution for the policy function, $k_{t+1} = \Phi(k_t, z_t)$. Couple of good sources for hints are Stokey, Prescott, and Lucas (1989, excercise 2.2, p. 12) and Sargent (1987, exercise 1.1, p. 47).

## Homework 2

For the model in section 3 of the notes consider the following functional forms:

$$u(c_t, \ell_t) = \ln\, c_t + a\, \ln\, (1 - \ell_t)$$

$$F(K_t, L_t, z_t) = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}$$

Write out all the characterizing equations for the model using these functional forms.

Can you use the same tricks as in homework 1 to solve for the policy function in this case? Why or why not?

## Homework 3

For the model in section 3 of the notes consider the following functional forms:

$$u(c_t, \ell_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + a \ln (1 - \ell_t)$$
$$F(K_t, L_t, z_t) = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}$$

Write out all the characterizing equations for the model using these functional forms.

## Homework 4

For the model in section 3 of the notes consider the following functional forms:

$$u(c_t, \ell_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + a \frac{(1 - \ell_t)^{1-\xi} - 1}{1-\xi}$$
$$F(K_t, L_t, z_t) = e^{z_t} \left[\alpha K_t^{\eta} + (1-\alpha) L_t^{\eta}\right]^{\frac{1}{\eta}}$$

Write out all the characterizing equations for the model using these functional forms.

## Homework & Lab 5a

For the model in section 3 of the notes consider the following functional forms:

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}$$
$$F(K_t, L_t, z_t) = K_t^{\alpha} (L_t e^{z_t})^{1-\alpha}$$

Write out all the characterizing equations for the model using these functional forms. Assume $\ell_t = 1$.

Write out the steady state versions of these equations. Solve algebraically for the steady state value of $k$ as a function of the steady state value of $z$ and the parameters of the model. Numerically solve for the steady state values of all variables using the following parameter values: $\gamma = 2.5$, $\beta = .98$, $\alpha = .40$, $\delta = .10$, $\bar{z} = 0$ and $\tau = .05$. Also solve for the steady state values of output and investment. Compare these values with the ones implied by the algebraic solution.

## Homework & Lab 5b

For the model in section 3 of the notes consider the following functional forms:

$$u(c_t, \ell_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + a\frac{(1-\ell_t)^{1-\xi} - 1}{1-\xi}$$
$$F(K_t, L_t, z_t) = K_t^{\alpha}(L_t e^{z_t})^{1-\alpha}$$

Write out all the characterizing equations for the model using these functional forms. Write out the steady state versions of these equations. Numerically solve for the steady state values of all variables using the following parameter values: $\gamma = 2.5$, $\xi = 1.5$, $\beta = .98$, $\alpha = .40$, $a = .5$, $\delta = .10$, $\bar{z} = 0$, and $\tau = .05$. Also solve for the steady state values of output and investment.

## Homework 6

For the steady state in section 3.7 of the notes use total differentiation to solve for the full set of comparative statics and sign them where possible. Find $\frac{\partial y}{\partial x}$ for $y \in \{\bar{k}, \bar{w}, \bar{r}, \bar{\ell}\}$ and $x \in \{\delta, \tau, \bar{z}\}$.

Assume $f_K > 0$, $f_{KK} < 0$, $f_L > 0$, $f_{LL} < 0$, $u_c > 0$, $u_{cc} < 0$, $u_\ell < 0$ and $u_{\ell\ell} > 0$.

Note this problem gets very tedious. Solving via MAPLE would be a wonderful shortcut and you are welcome to proceed that way. However, you may also just shown how you could set the problem up using linear algebra and show what steps need to be taken to get the comparative statics, but not actually perform the algebra.

## Lab 7a

Set up a discrete grid for $K$ with 100 values ranging from .0001 to $5\bar{K}$. Also set up a discrete grid for $z$ with 100 values ranging from $-5\sigma$ to $+5\sigma$. Set up a value function array, $V$ that stores the value for all 10,000 possible permuations of $K$ and $z$. Also set up a policy function array, $H$, that stores the optimal index value of $K'$ for all all 10,000 possible permuations of $K$ and $z$.

To begin assume that all elements of $V$ are zero and that all elements of $H$ point to the lowest possible value for $K$ (.0001).

Loop over all possible values of $K$ and $z$ and for each combination find 1) the optimal value of $K'$ from the 100 possible values. Store this value in an updated policy function array, $H_{new}$. Also find 2) the value implied by this choice given the current value function. Store this in an updated value function array, $V_{new}$.

Once this is completed for all $K$ and $z$ check to see if $V$ is approprimately equal to $V_{new}$. If so, output the value function and policy function arrays. If not, replace $V$ with $V_{new}$ and $H$ with $H_{new}$ and repeat the search above.

When finished plot the three-dimensional surface plot for the policy function $K' = H(K, z)$. Compare this with the closed form solution from the notes.

## Lab 7b

Repeat the above exercise using $k \equiv \ln K$ in place of $K$ as the endogenous state variable.

## Lab 8a

Using Uhlig's notation analytically find the values of the following matrices: $F, G, H, L, M$ & $N$ as functions of the parameters. Given these find the values of $P\&Q$, also as functions of the parameters. Imposing our calbrated parameter values, plot the three-dimensional surface plot for the policy function $K' = H(K, z)$. Compare this with the closed form solution fromthe notes and the solution you found using the grid search method.

## Lab 8b

Repeat the above exercise using $k \equiv \ln K$ in place of $K$ as the endogenous state variable.

## Lab 9

Numerically, find $\frac{\partial y}{\partial x}$ for $y \in \{\bar{k}, \bar{w}, \bar{r}, \bar{\ell}\}$ and $x \in \{\delta, \tau, \bar{z}\}$ assuming $u(c_t, \ell_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma} + a\frac{(1-\ell_t)^{1-\xi}-1}{1-\xi}$ and $F(K_t, L_t, z_t) = K_t^{\alpha}(L_t e^{z_t})^{1-\alpha}$. Use the following parameter values: $\gamma = 2.5$, $\xi = 1.5$, $\beta = .98$, $\alpha = .40$, $a = .5$, $\delta = .10$, $\bar{z} = 0$, and $\tau = .05$.

## Homework 10

Do the necessary tedious matrix algebra necessary to transform equation (6.5) into equation (6.8).

## Lab 11

Assume $n_X = n_Z = 1$. Show how to solve for the elements of $P$ and $Q$. Write a program in Python that will do this given $F, G, H, L, M$ and $N$ as inputs.

Now assume $n_X$ and $n_Z$ take on arbitrary values greater than one. Show how to solve for the elements of $P$ and $Q$. Write a program in Python that will do this given $F, G, H, L, M$ and $N$ as inputs.

The MATLAB programs for Uhlig (1999) are available online and may be of some help structuring the Python code. The discussion in the paper itself may also be a big help.

## Homework & Lab 12

Suppose that instead of including the jump variables of our our model in the vector $X_t$ above, we separated them into a separate vector, $Y_t$. In this case we can linearize our system into the following equations:

$$
\begin{aligned}
0 &= A\tilde{X}_t + B\tilde{X}_{t-1} + C\tilde{Y}_t + D\tilde{Z}_t \\
0 &= E_t\left\{F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + J\tilde{Y}_{t+1} + K\tilde{Y}_t + L\tilde{Z}_{t+1} + M\tilde{Z}_t\right\} \\
\tilde{Z}_t &= N\tilde{Z}_{t-1} + \varepsilon_t
\end{aligned}
$$

Assume that $C$ is of full rank. By once again hypothesizing that the transition functions for the model are log-linear, that is they are of the form:

$$
\begin{aligned}
\tilde{X}_t &= P\tilde{X}_{t-1} + Q\tilde{Z}_t \\
\tilde{Y}_t &= R\tilde{X}_{t-1} + S\tilde{Z}_t
\end{aligned}
$$

Derive algebraic solutions for $P$, $Q$, $R$, and $S$ using techniques similar to those in the notes. Modify the program you wrote in Homework 9 slightly to accommodate separating jump variables from state variables. What advantages might this approach have computationally? In setting up the matricies describing the model?

## Lab 13

Assume $n_X = n_Z = 1$. Write a Python program that will find the numerical values for the derivative of the characterizing equation, taking the parameters as given. Note that you will need to find the steady state value of $X$ first.

Now assume $n_X$ and $n_Z$ take on arbitrary integer values greater than one. Write a Python program that will find the numerical values for the derivatives of the characterizing equations in this case.

## Lab 14

For the log-linearized model in section 6.6 of the notes consider the following fundamental functional forms:

$$u(c_t, \ell_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + a\frac{(1-\ell_t)^{1-\gamma} - 1}{1-\gamma}$$

$$F(K_t, L_t, z_t) = K_t^{\alpha}(L_t e^{z_t})^{1-\alpha}$$

Use the following paramter values: $\gamma = 2.5$, $\beta = .98$, $\alpha = .40$, $a = .5$, $\delta = .10$, $\bar{z} = 0$, $\rho_z = .9$ and $\tau = .05$.

Find the values of $P$ and $Q$ in this case if $X_t = \{k_{t-1}, \ell_t\}$.

## Lab 15

Use the law of motion and approximate policy function from homework 14 to generate 1000 artificial time series for an economy where each time series is 250 periods long. Start each simulation off with a starting value for $k$ that is ten percent below the steady state value, and a value of $z$ that is one standard deviation below zero.

Use $\sigma_z^2 = .004$.

For each simulation save the time-series for GDP, consumption and investment. When all 1000 simulations have finished generate a graph for each of these time-series showing the average value over the simulations for each period, and also showing the five and ninety-five percent confidence bands for each series each period.

In addition, calcuate the mean, standard deviation, autocorrelation and correlation with output for each series over each simulation and report the average values and standard deviations for these moments over the 1000 simulations.

### Lab 16

Using the same setup as homework 14, generate impulse response functions for GDP, consumption and investment with lags from zero to forty periods.

## References


Sargent, T. J. (1987): *Dynamic Macroeconomic Theory*. Harvard University Press, Cambridge, Massachusetts.

Stokey, N. L., E. C. Prescott, and R. E. Lucas (1989): *Recursive methods in economic dynamics*. Harvard University Press, Cambridge, Massachusetts.

Uhlig, H. (1999): "A toolkit for analyzing nonlinear dynamic stochastic models easily," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, pp. 30–61. Oxford University Press.