Econ 413R: Computational Economics

Spring Term 2013

Perturbation Methods for DSGE Models
Homework Set

Week 3

Homework 1

For the function $F(k', k) = (k^{.35} + .9k - k')^{-2.5} - .95(k'^{.35} + .9k')^{-2.5} = 0$, use perturbation methods to find the cubic approximation of $k' = f(k)$ about the point $k = 0.1$. In this case, $k' = f(0.1) = 0.069986$.

Homework 2a

For the Brock and Mirman model with the default parameter values find the scalar values of H_X, H_X, H_{XX}, H_{XZ}, H_{ZZ} and $H_{\sigma\sigma}$.

Plot the three-dimensional surface plot for the policy function $K' = H(K, z)$. Compare this with the closed form solution from the notes and the two approximations from the previous homework set (numbers 7a and 8a).

Homework 2b

Repeat the above exercise using $k \equiv \ln K$ in place of K as the endogenous state variable.

Lab 3a

Using Dynare replicate the results from the previous homework set, problem 14. Be sure to specify a first-order approximation. Report the linear coefficents in the policy function. Then replicate the moments and IRFs from problems 15 and 16.

Lab 3b

Repeat the above exercise using a second-order approximation of the policy function. Report all linear and quadratic coefficients. Comment on any differences.

Lab 3c

Repeat problem 3a using a third-order approximation of the policy function. Report all linear, quadratic and cubic coefficients. Comment on any differences.

Exercises

Homework 1

Using the methods discussed in section 3.1.1, write a recursive form of the infinite sum, S_t.

Derive the Euler equation.

Homework 2

Using the methods discussed in section 3.1.2, write a recursive form of the infinite sum, D_t.

Homework 3

Using the same setup as Problem 14 from the Week 3 DSGE homework, let's tweak the utility function a little bit by including a preference shock to consumption Lester, Pries, and Sims (2013).

$$u(c_t, \ell_t) = \nu_t \frac{c_t^{1-\gamma}}{1-\gamma} + a\frac{(1-\ell_t)^{1-\gamma}}{1-\gamma}$$

where $\nu_t = (1-\rho_\nu) + \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$; with $\varepsilon_t^\nu \sim$ i.i.d.$(0, \sigma_\nu^2)$ and $\sigma_\nu = .02$.

nu_t is a multiplicative shock to utility from consumption and will therefore shock marginal utility of consumption as well. It is like a demand shock. Note that the coefficient of relative risk aversion and the elasticity of substitution are independent of this shock.

Also assume $\varepsilon_t^z \sim$ i.i.d.$(0, \sigma_z^2)$ and $\sigma_z = .01$.

Rewrite the two characterizing equations as they should now appear with this preference shock to consumption.

Write the Dynare code to perform a stochastic simulation of this model for 2100 periods, use the second-order Taylor-series approximation, and generate IRFs of all the variables to each shock for 100 periods. Provide the output, graphs and code as part of your homework submission. Comment on the IRF graphs; intuitively describe the responses of output and consumption each of the shocks.

Homework 4

Modify the Dynare code used above, eliminating the stochastic simulation, and gearing it toward a Bayesian estimation of the following parameters: $\gamma, a, \beta, \rho_z, \rho_\nu, \sigma^z$ and σ_ν. You will need to include new variables in your model that are more suited toward Bayesian estimation:

$$dc = c_t - c_{t-1}$$

$$dy = y_t - y_{t-1}$$

These will be the variables you will be observing in the code. I will provide the data file for you. Start your estimation according to the table below.[1]

Do this estimation for 20000 replications, 3 Metropolis Hastings blocks, drop the first 15% of the replications (burn in of 15%), start `mh_jscale` at 0.5 and ADJUST the `mh_jscale` as you monitor the acceptance rate so that you get an acceptance rate as close to 25% as possible once the replications within

[1] Prior distributions inspired by the in class example and Del Negro, Schorfheide, Smets and Wouters, 2005

Table 1: Bayesian Estimation Setup

Parameter	**Prior Distribution**	**Prior Mean**	**Prior St. Dev.**
γ	Gamma	2.5	0.05
a	Beta	0.5	0.01
β	Beta	0.98	0.002
ρ_z	Beta	0.90	0.05
ρ_ν	Beta	0.95	0.05
σ_z	Inverse Gamma	0.01	∞
σ_ν	Inverse Gamma	0.02	∞

the block seem to stabilize around a particular rate. Report the posterior mode for each of the parameters. This is the most important information. Also report the posterior mean and confidence intervals for each parameter from the provided output. Provide the graphs from the output.

If you were to redo the Bayesian estimation, for which parameters would you re-specify the prior distribution or how would you modify the replication procedure? Use the graphs as support for your answer. If you feel the priors were reasonably specified, provide support (again from the graphs) toward the legitimacy of the priors specified.

References

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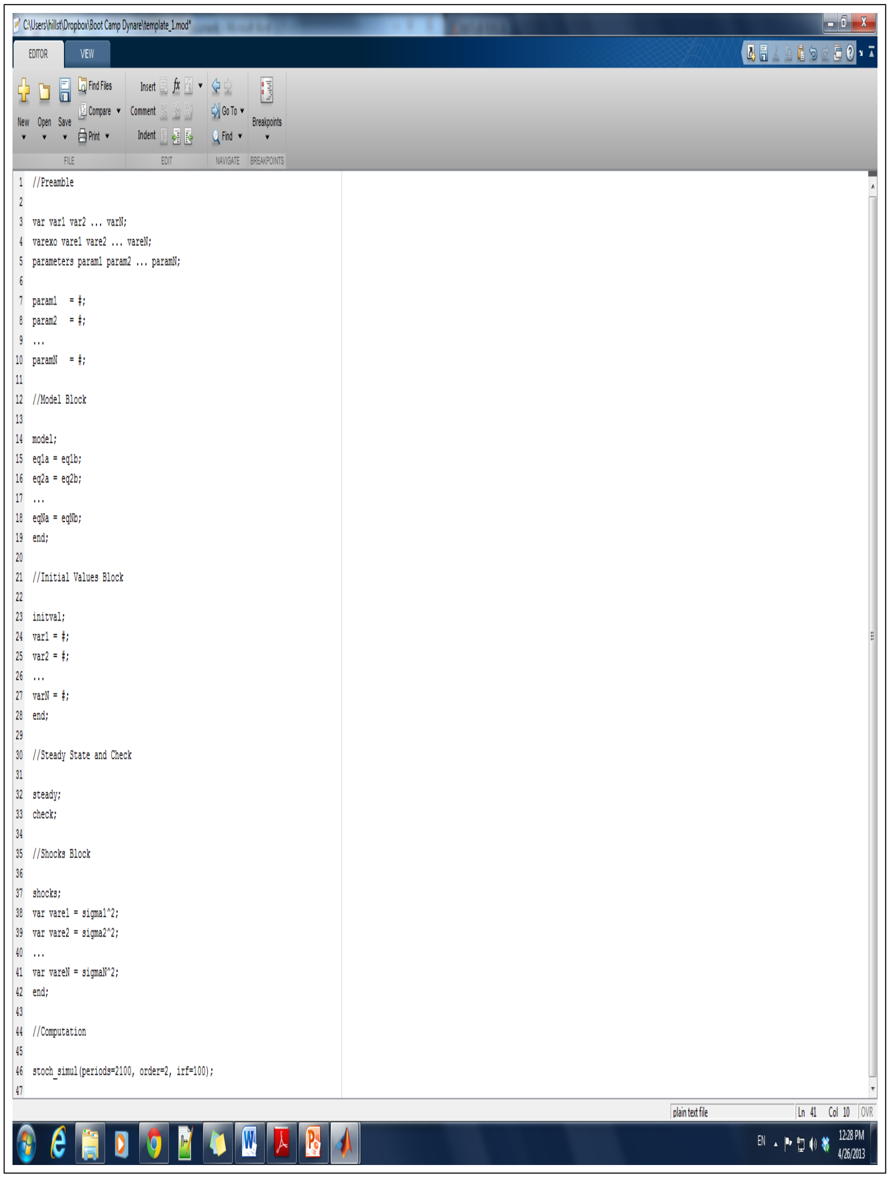
Figure 1: Sample Dynare Model File



4 Exercises

Exercise 1. Estimate the four parameters of the Brock and Mirman (1972) model $(\alpha, \beta, z_L, z_H)$ described by equations (2) through (8) by SMM. Choose the four parameters to match the following six moments from the 66 periods of empirical data $\{Y_t, k_t, c_t\}_{t=1}^{66}$ in `smmdata.txt`: $\text{mean}(Y_t)$, $\text{mean}(c_t)$, $\text{var}(Y_t)$, $\text{var}(c_t)$, $\text{corr}(k_t, Y_t)$, and $\text{corr}(k_t, k_{t+1})$. In your simulations of the model, set $T = 66$ and $S = 10,000$. Start each of your simulations from $k_1 = \text{mean}(k_t)$ from the `smmdata.txt` file. Use the `scipy.optimize.minimize` constrained minimizer command with the method set to `method='TNC'` and the tolerance set to `tol=1e-10`. Input the bounds to be $\alpha, \beta \in [\varepsilon, 1-\varepsilon]$, $z_L \in [-2, 0]$, and $z_H \in [1, 3]$, and where $\varepsilon = 1e-10$. Report your solution $(\hat{\alpha}, \hat{\beta}, \hat{z}_L, \hat{z}_H)$, the vector of moment differences, the sum of squared moment differences, and the computation time.

Once you have successfully estimated the parameters of a model by SMM, the question remains of how good the estimates $\hat{\theta}_{SMM}$ are. How can you check the accuracy? The first way is to see how close your simulated average of your model moments came to their target empirical moments that you were trying to match. However, this is not sufficient because you chose the parameters to minimize that distance. The best SMM estimations match well the moments that they used for the estimation, and they match some important moments that were not used in the estimation. These "outside" moments are a key piece of evidence that your model and estimation are good.

References

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