4 Exercises

Exercise 1. Estimate the four parameters of the Brock and Mirman (1972) model $(\alpha, \beta, z_L, z_H)$ described by equations (2) through (8) by SMM. Choose the four parameters to match the following six moments from the 66 periods of empirical data $\{Y_t, k_t, c_t\}_{t=1}^{66}$ in `smmdata.txt`: mean(Y_t), mean(c_t), var(Y_t), var(c_t), corr(k_t, Y_t), and corr(k_t, k_{t+1}). In your simulations of the model, set $T = 66$ and $S = 10,000$. Start each of your simulations from $k_1 = \text{mean}(k_t)$ from the `smmdata.txt` file. Use the `scipy.optimize.minimize` constrained minimizer command with the method set to `method='TNC'` and the tolerance set to `tol=1e-10`. Input the bounds to be $\alpha, \beta \in [\varepsilon, 1-\varepsilon]$, $z_L \in [-2, 0]$, and $z_H \in [1, 3]$, and where $\varepsilon = 1e-10$. Report your solution $(\hat{\alpha}, \hat{\beta}, \hat{z}_L, \hat{z}_H)$, the vector of moment differences, the sum of squared moment differences, and the computation time.

Once you have successfully estimated the parameters of a model by SMM, the question remains of how good the estimates $\hat{\theta}_{SMM}$ are. How can you check the accuracy? The first way is to see how close your simulated average of your model moments came to their target empirical moments that you were trying to match. However, this is not sufficient because you chose the parameters to minimize that distance. The best SMM estimations match well the moments that they used for the estimation, and they match some important moments that were not used in the estimation. These "outside" moments are a key piece of evidence that your model and estimation are good.

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